hep-th/0601164

Kappa symmetry, generalized calibrations and spinorial geometry

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Abstract

We extend the spinorial geometry techniques developed for the solution of supergravity Killing spinor equations to the kappa symmetry condition for supersymmetric brane probe configurations in any supergravity background. In particular, we construct the linear systems associated with the kappa symmetry projector of M- and type II branes acting on any Killing spinor. As an example, we show that static supersymmetric M2-brane configurations which admit a Killing spinor representing the $SU(5)$ orbit of $Spin(10,1)$ are generalized almost hermitian calibrations and the embedding map is pseudo-holomorphic. We also present a bound for the Euclidean action of M- and type II branes embedded in a supersymmetric background with non-vanishing fluxes. This leads to an extension of the definition of generalized calibrations which allows for the presence of non-trivial Born-Infeld type of fields in the brane actions.

The supersymmetry condition for a probe brane propagating in a supersymmetric supergravity background with a Killing spinor ϵ is

$$\Gamma\epsilon = \epsilon \ , \tag{0.1}$$

where Γ is the kappa symmetry projector, $\Gamma^2 = 1$ and $\mathrm{tr}\,\Gamma = 0$. This condition was found for M2-branes in [1], extended to Euclidean M-branes and type II NS$\otimes$NS branes in [2] and generalized to D-branes with non-vanishing worldvolume fluxes in [3]. The projector Γ is an endomorphism of a Clifford algebra and depends on the embedding map X of the brane world-volume into spacetime and on the spacetime frame e [4]-[9]. For some branes, it also depends on other world-volume fields and on some of the spacetime gauge potentials, e.g. for D-branes it depends on the Born-Infeld field F and on the NS$\otimes$NS two-form gauge potential B and so $\Gamma = \Gamma(e, X, F, B)$. The kappa symmetry projectors that we use in this paper are summarized in [3]. Suppose that a supergravity background admits N Killing spinors, $\{\epsilon_i; i = 1, \dots, N\}$, $\mathcal{D}\epsilon_i = 0$, $\mathcal{A}\epsilon_i = 0$, where $\mathcal{D}$ is the supercovariant connection associated with the gravitino supersymmetry transformation and $\mathcal{A}$ are algebraic conditions associated with the supersymmetry transformations of the rest of the fermions. The solutions of the kappa symmetry condition can be written as $\epsilon_r = \sum_i u_{ri}\epsilon_i$, where u are real constant coefficients. Supersymmetric brane configurations are those for which (0.1) admits as solutions K, $0 < K \leq N$, linearly independent Killing spinors. In such a case, the brane probe preserves K spacetime supersymmetries.

In [10], a spinorial geometry method has been proposed to solve the Killing spinor equations of supergravity theories. This method turns the Killing spinor equations to a linear system of algebraic equations and a parallel transport equation which however do not contain spacetime gamma matrices. The linear system can be solved to determine some of the fluxes in terms of the geometry and to find the conditions on the geometry of spacetime imposed by supersymmetry. The advantage of the method is that the calculation can be done rather efficiently and in generality, i.e. without making ansatze for the spacetime geometry, supergravity fluxes or Killing spinors. The spinorial geometry is based on the following ingredients:

- a description of spinors in terms of forms,
- an oscillator basis in the space of spinors,
- the use of a gauge symmetry of the Killing spinor equations to bring the Killing spinors into a normal form .

The latter is a key ingredient because it can be used to solve the Killing spinor equations rather efficiently for a small number of supersymmetries and simplifies the linear systems in all cases.

Although the spinorial geometry method has been initially developed to solve the supergravity Killing spinor equations, it can be easily adapted to other spinorial problems and in particular to solve the superymmetry condition for brane probes (0.1) without using ansatze for the fields. The first two ingredients of the method are based on well known properties of spinors and they have been extensively explained and applied in the context of ten- and eleven-dimensional supergravities [10]-[14], so we shall not elaborate

upon these here. It remains to find the gauge symmetry of the supersymmetry condition (0.1) which can be used in the context of spinorial geometry. These are defined as those local transformations, automorphisms of the spinor bundle, with values in $GL(\Delta)$, Δ is a spinor representation[1], that act on the spinor ϵ and leave the form of the supersymmetry condition invariant, i.e.

$$g^{-1}\Gamma(e, X, \mathcal{F})g = \Gamma(e^g, X^g, \mathcal{F}^g) \tag{0.2}$$

where $\mathcal{F}$ denotes collectively the remaining fields that are required for the definition of Γ and $e \to e^g$, $X \to X^g$ and $\mathcal{F} \to \mathcal{F}^g$ is an induced transformation on the fields. It is well-known that Γ is invariant under worldvolume orientation preserving diffeomorphisms. However the gauge transformations that are relevant in the context of spinorial geometry are the local $Spin \subset GL$ transformations. In particular for M-branes, the gauge group is $Spin(10,1)$ which is the same as the gauge group of the Killing spinor equations of eleven-dimensional supergravity. For both M2- and M5-branes, the spin transformations $g \in Spin(10,1)$ induce Lorentz rotations on the spacetime frame e. This can be easily seen by a direct inspection of the kappa symmetry projectors of the M2-and M5-branes, in the form given in [5] and in [6], respectively, see also (0.3).

An inspection of the kappa symmetry projectors of type II branes [4, 1, 5, 7] reveals that the gauge symmetry of the kappa symmetry projectors is $Spin(9,1)$. This is again compensated by a Lorentz transformation of the spacetime frame. This gauge symmetry is the same as that of the Killing spinor equations of the associated supergravity theories[2]. Therefore gauge symmetry of the supergravity Killing spinor equations relevant to the spinorial geometry leaves invariant the associated brane probe supersymmetry conditions (0.1). This can be used to simultaneously solve the supergravity Killing spinor equations and the brane supersymmetry conditions. We shall use this in the M2-brane example that we shall present below.

Next we shall explain the systematics of the supersymmetry condition (0.1), i.e. the way to turn (0.1) into a linear system which does not contain spacetime gamma matrices for any spinor ϵ. Let us first consider the M-branes. The kappa symmetry projector of the M2-brane is

$$\Gamma_{M2} = \frac{1}{3!}\frac{1}{\sqrt{|\det(\gamma)|}}\epsilon^{\mu_1\mu_2\mu_3}\partial_{\mu_1}X^{M_1}\partial_{\mu_2}X^{M_2}\partial_{\mu_3}X^{M_3}e^{A_1}_{M_1}e^{A_2}_{M_2}e^{A_3}_{M_3}\Gamma_{A_1A_2A_3} \tag{0.3}$$

where μ_1, μ_2, μ_3 are worldvolume indices, M_1, M_2, M_3 are spacetime coordinate indices and A_1, A_2, A_3 are spacetime frame indices. In addition, Γ_A are the spacetime gamma matrices, $\Gamma_{A_1A_2A_3}$ denotes the skew-product of three gamma matrices, and $\gamma_{\mu\nu} = \partial_\mu X^M \partial_\nu X^N g_{MN}$ is the induced metric on the brane world-volume by pulling back the spacetime metric g with the brane embedding map X. The kappa symmetry projectors of branes without other worldvolume and spacetime fields are similarly constructed. The above kappa

[1] The spinor representation Δ depends on the supergravity theory under investigation.

[2] In fact the gauge symmetry of IIB Killing spinor equations is $Spin(9,1) \times U(1)$ but the kappa symmetry projector Γ for the D-branes as written in e.g. [5] preserves only $Spin(9,1)$ and this suffices for our purpose. Furthermore the IIA supergravity Killing spinor equations have a hidden $Spin(10,1)$ invariance because they are related to the eleven-dimensional supergravity ones by dimensional reduction. However in terms of ten-dimensional variables only the $Spin(9,1)$ gauge group is manifest.

symmetry projector can be written in short-hand notation as

$$\Gamma_{M2} = \Phi_{ABC}\Gamma^{ABC} = \Phi_{(3)}\Gamma^{(3)} \ . \tag{0.4}$$

Similarly, the M5-brane kappa symmetry projector in the form given in [6] can be written as

$$\Gamma_{M5} = \Phi_{(2)}\Gamma^{(2)} + \Phi_{(3)}\Gamma^{(3)} + \Phi_{(4)}\Gamma^{(4)} + \Phi_{(5)}\Gamma^{(5)} \ , \tag{0.5}$$

where we have used Poincaré duality to relate forms with degree $11-k$ to forms of degree k, $k<6$, and their associated Clifford algebra elements. If the M5-brane worldvolume three-form field strength vanishes, then $\Phi_{(2)} = \Phi_{(3)} = \Phi_{(4)} = 0$ and so $\Gamma_{M5} = \Phi_{(5)}\Gamma^{(5)}$.

In the context of eleven-dimensional supergravity, one can expand the spinors in a time-like or a null oscillator basis depending on whether one investigates Killing spinors representing the orbit $SU(5)$ or the orbit $(Spin(7) \ltimes \mathbb{R}^8) \times \mathbb{R}$ of $Spin(10,1)$ in the spinor representation $\Delta_{\mathbf{32}}$ [15, 16]. The choice between these two bases is made purely for convenience as the final result is independent from the basis used. To construct the linear system associated with the supersymmetry conditions for the M-branes in the time-like basis, as for the supergravity Killing spinor equations, one separates the gamma matrix Γ_0 along the time-direction from the rest. In such case, the kappa symmetry projectors of the M-branes can be written as

$$\Gamma_{Mp} = \sum_k \Psi_{(k)}\tilde{\Gamma}^{(k)} + \sum_k \mathcal{X}_{(k)}\tilde{\Gamma}^{(k)}\Gamma_0 \tag{0.6}$$

summing over appropriately chosen k, e.g. for the M2-brane we have

$$\Gamma_{M2} = \Psi_{(3)}\tilde{\Gamma}^{(3)} + \mathcal{X}_{(2)}\tilde{\Gamma}^{(2)}\Gamma_0 \ , \tag{0.7}$$

where $\tilde{\Gamma}^{(k)}$ denote skew-symmetric k-products of the remaining ten Γ^i, $i = 1,\dots,10$, gamma matrices. A $Spin(10,1)$ Majorana Killing spinor can be written in terms of forms as

$$\begin{aligned}\epsilon \ &= \ f(1+e_{12345}) + ig(1-e_{12345}) + \sqrt{2}u^i(e_i + \frac{1}{4!}\epsilon_i{}^{jklm}e_{jklm}) + i\sqrt{2}v^i(e_i - \frac{1}{4!}\epsilon_i{}^{jklm}e_{jklm}) \\ &\quad + \frac{1}{2}w^{ij}(e_{ij} - \frac{1}{3!}\epsilon_{ij}{}^{klm}e_{klm}) + \frac{i}{2}z^{ij}(e_{ij} + \frac{1}{3!}\epsilon_{ij}{}^{klm}e_{klm}) \ ,\end{aligned} \tag{0.8}$$

where f, g, u^i, v^i, w^{ij} and z^{ij} are real spacetime functions. We use the spinor conventions of [12]. Since Γ_{Mp} is a linear operator to compute $\Gamma_{Mp}\epsilon$, it suffices to evaluate $\Gamma_{Mp}e_{i_1\dots i_I}$ on the six types of spinors $e_{i_1\cdots i_I}$ with $I = 0,\dots,5$. In particular, we have

$$\Gamma_{Mp}e_{i_1\dots i_I} = \sum_k \Psi_{(k)}\tilde{\Gamma}^{(k)}e_{i_1\dots i_I} + i(-1)^I \sum_k \mathcal{X}_{(k)}\tilde{\Gamma}^{(k)}e_{i_1\dots i_I} \ . \tag{0.9}$$

The expansion of $\tilde{\Gamma}^{(k)}e_{i_1\dots i_I}$ for $0 \le k < 6$ in terms of the holomorphic "canonical" basis $\{1, \Gamma^{\bar{\alpha}}1, \dots, \Gamma^{\bar{\alpha}_1\bar{\alpha}_2\bar{\alpha}_3\bar{\alpha}_4\bar{\alpha}_5}1\}$, $\alpha_1,\dots,\alpha_5 = 1,\dots,5$, can be read either from the universal formulae or from the results that have been presented in the appendices of [12].

In the null basis the analysis is similar. In this case, one separates the gamma matrix along the tenth spatial direction denoted by $\Gamma_\natural = -\Gamma_0\Gamma_1\cdots\Gamma_9$ from the rest, see [12]. The kappa symmetry projection operators of the M-branes can be written as

$$\Gamma_{Mp} = \sum_k \Psi_{(k)}\tilde{\Gamma}^{(k)} + \sum_k \mathcal{X}_{(k)}\tilde{\Gamma}^{(k)}\Gamma_\natural \ , \tag{0.10}$$

where now $\tilde{\Gamma}^{(k)}$ denotes the shew-symmetric k-products of the remaining ten, Γ^i, $i = 0, \dots, 9$, gamma matrices. Again it suffices to evaluate Γ_{Mp} on the basis $e_{i_1\dots i_I}$ for $0 \leq k < 6$. In particular, one finds that

$$\Gamma_{Mp}e_{i_1\dots i_I} = \sum_k \Psi_{(k)}\tilde{\Gamma}^{(k)}e_{i_1\dots i_I} + (-1)^{I+1}\sum_k \mathcal{X}_{(k)}\tilde{\Gamma}^{(k)}e_{i_1\dots i_I} \ . \tag{0.11}$$

The expansion of $\tilde{\Gamma}^{(k)}e_{i_1\dots i_I}$ for $0 \leq k < 6$ in terms of the "canonical" null oscillator basis $\{1, \Gamma^{\bar{a}_1}1, \dots, \Gamma^{\bar{a}_1\dots\bar{a}_5}1\}$, $\bar{a}_1, \dots, \bar{a}_5 = +, 1, \dots, 4$ can be read from the universal formulae of [12].

The construction of the linear system associated with the supersymmetry condition (0.1) for type II branes can be done in a way similar to that for the M-branes we have explained above. The Majorana-Weyl spinors, $\Delta^{\pm}_{\mathbf{16}}$, of $Spin(9,1)$ can be expanded in a null oscillator basis. The most general Killing spinor of IIB supergravity can be written as

$$\epsilon = p1 + qe_{1234} + u^i e_{i5} + \frac{1}{2}v^{ij}e_{ij} + \frac{1}{6}w^{ijk}e_{ijk5} \ , \tag{0.12}$$

where p, q, u, v and w are complex functions on the spacetime, and $i, j, k = 1, 2, 3, 4$. Our spinor conventions and the construction of the null basis can be found in [13]. The kappa symmetry projectors of IIB branes, which we denote collectively by Γ_{IIB}, can be expanded as

$$\Gamma_{IIB} = \sum_k \Phi_{(k)}\Gamma^{(k)} \ , \tag{0.13}$$

where $\Phi_{(k)}$ may also carry internal indices, e.g. $SL(2,\mathbb{R})$ indices for type IIB D-branes. After an analysis similar to that we have presented for the M-branes, the evaluation of $\Gamma_{IIB}\epsilon$ for ϵ given in (0.12) reduces to the evaluation of $\Gamma^{(k)}\sigma_I$, $0 \leq k \leq 5$, where σ_I are the five types of spinors $1, e_{1234}, e_{i5}, e_{ij}$ and e_{ijk5}. Again, the expressions for $\Gamma^{(k)}\sigma_I$ in terms of the "canonical" null oscillator basis $\{1, \Gamma^{\bar{a}_1}1, \dots, \Gamma^{\bar{a}_1\dots\bar{a}_5}1\}$, $\bar{a}_1, \dots, \bar{a}_5 = +, 1, \dots, 4$ can be either be computed from the universal formulae or can be read from the appendices of [13].

Next let us turn to supersymmetry conditions for type IIA branes. The Killing spinors of IIA supergravity can be written as

$$\begin{aligned} \epsilon \ &= \ f1 + ge_{1234} + u^i e_{i5} + \frac{1}{2}v^{ij}e_{ij} + \frac{1}{6}w^{ijk}e_{ijk5} \\ &\qquad + \tilde{f}e_5 + \tilde{g}e_{12345} + \tilde{u}^i e_i + \frac{1}{2}\tilde{v}^{ij}e_{ij5} + \frac{1}{6}\tilde{w}^{ijk}e_{ijk} \ , \end{aligned} \tag{0.14}$$

where the components f, g, u, v, w and $\tilde{f}, \tilde{g}, \tilde{u}, \tilde{v}, \tilde{w}$ are real spacetime functions. The kappa symmetry projectors of type IIA branes, which we denote collectively by Γ_{IIA}, can be expanded as

$$\Gamma_{IIA} = \sum_k \Phi_{(k)} \Gamma^{(k)} \ . \tag{0.15}$$

To evaluate $\Gamma_{IIA}\epsilon$, where ϵ is given in (0.14), suffices to evaluate $\Gamma^{(k)}\sigma_I$ and $\Gamma^{(k)}\Gamma_5\sigma_I$, where σ_I denotes the five types of spinors as in the IIB case above. The expression for $\Gamma^{(k)}\sigma_I$ can be computed as in the IIB case. The expressions for $\Gamma^{(k)}\Gamma_5\sigma_I$ can be found from those of $\Gamma^{(k)}\sigma_I$ after exchanging the light-cone directions $- \leftrightarrow +$.

To illustrate the use of the spinorial geometry method to solve (0.1), consider a M2-brane probe propagating in a supersymmetric supergravity background preserving at least one supersymmetry. Other examples will be presented elsewhere [17]. In addition, let us assume that the Killing spinor that satisfies (0.1) has stability subgroup $SU(5)$ in $Spin(10,1)$. Since as we have explained both the Killing spinor equations of eleven-dimensional supergravity and the supersymmetry condition (0.1) are invariant under $Spin(10,1)$, one can choose ϵ to be any representative of the $SU(5)$ orbit in $\Delta_{\mathbf{32}}$. In particular, one can choose ϵ as in [10], i.e.

$$\epsilon = f(1 + e_{12345}) \ , \tag{0.16}$$

where f is a spacetime function. In turn, the supersymmetry condition (0.1) reads

$$\Gamma_{M2}(1 + e_{12345}) = 1 + e_{12345} \ . \tag{0.17}$$

Since the Killing spinor represents the $SU(5)$ orbit of $Spin(10,1)$ in $\Delta_{\mathbf{32}}$, it is convenient to use the time-like basis to do the analysis. Separating the gamma matrix Γ_0 from the rest and expanding (0.17) in the canonical holomorphic spinor basis $\{1, \Gamma^{\bar{\alpha}}1, \dots, \Gamma^{\bar{\alpha}_1\bar{\alpha}_2\bar{\alpha}_3\bar{\alpha}_4\bar{\alpha}_5}1\}$, $\alpha_1, \dots, \alpha_5 = 1, \dots, 5$, we find that

$$\begin{aligned} -2i\mathcal{X}^{\alpha}{}_{\alpha} &= 1 \\ \Psi^{\bar{\beta}\alpha}{}_{\alpha} &= 0 \ , \\ \Psi^{\gamma_1\gamma_2\gamma_3}\, \tilde{\epsilon}_{\gamma_1\gamma_2\gamma_3}{}^{\bar{\alpha}\bar{\beta}} + 2i\mathcal{X}^{\bar{\alpha}\bar{\beta}} &= 0 \ , \end{aligned} \tag{0.18}$$

where $\tilde{\epsilon} = \sqrt{2}\,\epsilon$ and ϵ is the holomorphic Levi-Civita tensor. In the analysis so far we have not used that ϵ is a Killing spinor. To continue, the spacetime metric of the eleven-dimensional background with Killing spinor $\epsilon = f(1 + e_{12345})$ can be written as

$$ds^2 = -f^4(dt + \alpha)^2 + ds^2_{10} \tag{0.19}$$

where $\partial/\partial t$ is a Killing vector field. The spacetime is a fibre bundle with fibre the orbits of $\partial/\partial t$ and base space an almost Hermitian manifold B with metric ds^2_{10} and compatible $SU(5)$ structure. In particular B admits a Hermitian form ω. The analysis of the geometry of eleven-dimensional backgrounds with one supersymmetry has originally been done in [18] using G-structures but here we follow the spinorial analysis of [10].

Next introduce the spacetime frame $e^0 = f^2(dt + \alpha)$ and $e^{\bar{\alpha}}, e^{\alpha}$, $\alpha = 1, \dots, 10$, and write $ds^2_{10} = 2\delta_{\alpha\bar{\beta}} e^{\alpha} e^{\bar{\beta}}$ and $\omega = -i\delta_{\alpha\bar{\beta}}\, e^{\alpha} \wedge e^{\bar{\beta}}$. Assuming that the M2-brane world-volume has topology $C \times \mathbb{R}$, where C is a two-dimensional surface, and that $\partial/\partial t$ is a rotation free Killing vector field, $d\alpha = 0$, which also leaves invariant the supergravity 4-form field strength, one can set $X^0 = t$, $\partial_\tau X^{\alpha} = 0$; τ is the worldvolume time coordinate. So we choose a static embedding such that $C \subset B$. Then, one can see that the only non-trivial conditions are

$$\begin{aligned} -2i\mathcal{X}^{\alpha}{}_{\underline{\alpha}} &= 1 , \\ \mathcal{X}^{\bar{\alpha}\bar{\beta}} &= 0 . \end{aligned} \tag{0.20}$$

The first equation can be rewritten as

$$\omega|_C = d\mathrm{vol}(C) , \tag{0.21}$$

where $\omega|_C$ is the hermitian form of B restricted on C and $d\mathrm{vol}(C)$ is the induced volume on C. This is precisely the condition expected from an almost hermitian generalized calibration [19, 20], i.e. that the restriction of the hermitian form on the cycle is equal to its volume. The second condition can be written as

$$dX^{\bar{\alpha}} \wedge dX^{\bar{\beta}} = 0 , \tag{0.22}$$

which is clearly satisfied iff the embedding map X is pseudo-holomorphic. Therefore the supersymmetry condition (0.1) specifies both the calibration type of the supersymmetric submanifolds of the spacetime and the embedding map X of the brane world-volume into the spacetime.

As we have seen in the example above, the equations that arise from the supersymmetry condition (0.1) are generalized calibrations. Initially the generalized calibrations where proposed as the supersymmetric cycles for branes embedded in backgrounds with non-vanishing gauge potentials that induce Wess-Zumino type of couplings in the world-volume brane actions [19]. In particular, world-volume fields, like the Born-Infeld field for D-branes, were required to vanish. Using generalized calibrations, one can establish a bound for the the energy of brane solitons. Extensions of the definition of the generalized calibrations have also been proposed to include other non-vanishing world-volume fields, like Born-Infeld type of fields, in some special cases, see e.g. [21, 22, 23]. However, it is clear that if one is interested in all the supersymmetric brane configurations, these are given by solving (0.1). So in the context of branes, one can define as generalized calibrated cycles the solutions of (0.1). In addition, as we have demonstrated using the spinorial geometry techniques, one can derive the conditions that these generalized calibrated cycles satisfy in all cases and without any additional restriction on either the world-volume or spacetime fields apart from those required by spacetime supersymmetry. One can also formulate a bound for the Euclidean action of branes in direct analogy to that of generalized calibrations mentioned above. We shall demonstrate this for Euclidean D-branes, see e.g. [2, 25, 26], and it easily extended to other cases. To see this, suppose that the kappa symmetry projector is hermitian $\Gamma^{\dagger} = \Gamma$ and $\Gamma^2 = 1$ and that the supersymmetry condition is given as in (0.1). We also write the (Euclidean) D-brane

action [24] as

$$S_{Dp} = \int_W d^{p+1}\sigma \sqrt{|\det(\gamma + \mathcal{F})|} + S_{WZ} \ , \tag{0.23}$$

where $\mathcal{F} = F - B$ and $S_{WZ} = \int_W C \wedge e^{\mathcal{F}}$ is the Wess-Zumino term. The kappa symmetry projector of [5] for D-branes can be written as

$$\Gamma = \frac{1}{\sqrt{|\det(\gamma + \mathcal{F})|}} \hat{\Gamma} \ , \tag{0.24}$$

where $\hat{\Gamma}$ is a Clifford algebra element that depends of the embedding maps, the Born Infeld field F and the pull back of B. Using a supersymmetry argument, define the spinor $\eta = \frac{1}{\sqrt{2}}(1 - \Gamma)\epsilon$ and observe that

$$0 \leq \eta^\dagger \eta = \epsilon^\dagger (1 - \Gamma)\epsilon \ . \tag{0.25}$$

Multiplying the above expression with $\sqrt{|\det(\gamma + \mathcal{F})|}$, assuming that it does not vanish, we find that

$$\sqrt{|\det(\gamma + \mathcal{F})|} \geq \frac{1}{\epsilon^\dagger \epsilon} \epsilon^\dagger \hat{\Gamma} \epsilon \ . \tag{0.26}$$

Integrating the above expression over the brane world-volume and adding S_{WZ}, we find[3] that

$$S_{Dp} \geq \int_W \phi + S_{WZ} \ , \quad \phi = \frac{1}{\epsilon^\dagger \epsilon} \epsilon^\dagger \hat{\Gamma} \epsilon \ . \tag{0.27}$$

Observe that ϕ is a (p+1)-form which depends on the spacetime Killing spinor bi-linears as well as the brane world-volume F, X and spacetime fields g and B. A similar bound for M5-brane solitons has been given in [21]. In the absence of worldvolume and spacetime fluxes, the bound (0.27) reduces to that of standard calibrations [27], see also [2]. Therefore it is tempting to identify ϕ with the calibration form of this type of generalized calibrations. In the context of generalized calibrations of [19], the generalized calibration form is related to the supergravity gauge potentials. If this applies here, then ϕ must be equal to the combination of the supergravity gauge potentials that appear in S_{WZ} of the Dp-brane action. This bound is attained, iff $\eta = 0$ and so (0.1) is satisfied.

Acknowledgments: G.P. thanks Ulf Gran, Jan Gutowski, and Diederic Roest for many helpful discussions on the spinorial geometry and C. Bachas for comments on calibrations.

[3]We have not normalized ϵ, as $\epsilon^\dagger \epsilon = 1$, because it is not apparent in the context of supergravity with fluxes that that the normalized spinor of a Killing spinor is Killing.

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